Simplified Takeoff Model for a Homebuilt Airplane

Russell E. Erb[*] and Karl B. Major[†]

United States Air Force Test Pilot School, Edwards AFB, CA 93523

A Matlab based simulation was previously created to model the takeoff performance of an Experimental Amateur Built Bearhawk. Since a device capable of running Matlab was not available in the cockpit and desiring a simpler model that could be realized in a spreadsheet for use on a smartphone or tablet instead of using a paper chart, a second model was created based on the partial differential methods of Ken Lush. These approximations were adjusted to provide results similar to the Matlab simulation. The residual errors between the Lush approximation and the Matlab simulation were analyzed using a Monte Carlo approach and found to be sufficiently small for operational use.

I. Background

In a previous paper "Takeoff Chart Development for a Homebuilt Airplane by Numerical Simulation" [1], a takeoff ground roll model was created for Experimental Amateur Built Bearhawk N6786E because no such performance data were provided as would be for a certificated airplane. This model ran in Matlab and calculated forces to find acceleration, which was numerically integrated to give speed and then ground roll. The model was validated by flight test results. The resulting model is shown in chart form in Fig. 1.
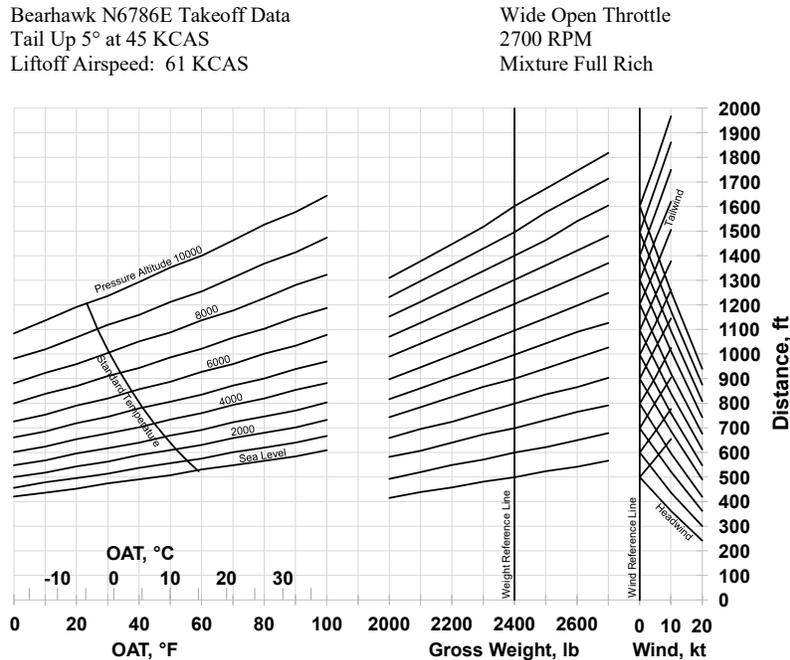
Bearhawk N6786E Takeoff Data
Tail Up 5° at 45 KCAS
Liftoff Airspeed: 61 KCAS

Wide Open Throttle
2700 RPM
Mixture Full Rich

Fig. 1 Bearhawk N6786E Takeoff Ground Roll Chart

---

[*] Aircraft Performance Master Instructor, Associate Professor, AIAA Senior Member

[†] Assistant Director of Operations, Associate Professor, AIAA Senior Member

The Matlab model provided a very sophisticated representation of the engine, propeller, and airframe, but it required a desktop or laptop computer to run. While a paper copy of Fig. 1 could be carried in the airplane and used as was done in the past, there was interest in a simpler approximation that could be run in a spreadsheet with just a few equations on an iPhone or iPad.

II. A Simpler Approximation

For this, we go way back to a method developed by Ken Lush before computers were available to run simulations such as the one run in Matlab. Lush's method was published in Ref. [2-3]. Lush's method uses partial derivatives with empirically collected coefficients to create ratios raised to exponents multiplied together to model effects of density, weight, and thrust. For this analysis, results showed that thrust effects could be ignored while retaining an acceptable prediction. Wind effects were derived by work-energy methods. The general methodology of Lush will be used, but the exponents will be adjusted to give a better match to the Matlab model. This result will be referred to as the Lush approximation.

A. Density Correction
The Lush correction for air density, represented by pressure altitude and temperature, is given by (Eqn. 6.328 [2])

$$S_{wt} = \frac{s_{std}}{\left(\frac{\sigma_s}{\sigma_t}\right)^{-2.4}} \tag{1}$$

where

s_{std}	ground roll for reference condition
S_{wt}	ground roll at reference gross weight, zero wind, corrected for density
σ_s	density ratio at reference condition
σ_t	density ratio at test condition

Strictly speaking, the equation above was for light aircraft with fixed pitch propellers, as used in Ref. [4]. A slightly different exponent may be specified for a constant speed propeller, but the important thing is the structure of the equation stays the same. For the Lush approximation, we will adjust the exponent to give the best fit to the Matlab model.

The reference conditions selected were:

Pressure Altitude	2000 feet
Outside Air Temperature	60°F
Gross Weight	2400 pounds
Headwind	0 knots
Ground Roll	630.5 feet (Matlab model result for these conditions)

These conditions were chosen as representative of Bearhawk operations at its home airport. The left set of curves (density correction) from Fig. 1 are shown in Fig. 2, along with a similar set of curves created by the Lush approximation above. As pointed out in Fig. 2, there is a mismatch between the two curve sets. Changing the exponent from -2.4 to -2.64 causes the curves to match at the reference condition of 2000 feet/60°F and at 10,000 feet/60°F, as shown in Fig. 3. Now both sets of curves match sufficiently within a "normal" temperature range of 50 to 75°F. As the equation only has one degree of freedom (changing the exponent), only limited adjustments can be made. Thus, we are stuck with the mis-matched slopes. Even so, the agreement is pretty good, not exceeding 40 feet at the extreme temperatures. At high temperatures, where runway length is more likely to be an issue, the Lush approximation predicts a longer takeoff roll. At low temperatures, the Lush approximation predicts a slightly shorter takeoff run. The Aircraft Owners and Pilots Association (AOPA) Air Safety Institute (ASI) recommends adding 50 percent to the Pilot's Operating Handbook takeoff distance over a 50-foot obstacle [5]. The shorter value predicted by the Lush approximation will be absorbed in the large pad that will be added to the final result, such as the 50 percent recommended by ASI.

As mentioned above, Lush's independent variables were density, weight, and thrust. Density and weight are clearly shown in Fig. 1 but thrust does not seem to be accounted for. In Lush's notes [3] at Equation 7-2, Lush shows that for propellers turning at maximum permissible speed during the takeoff roll the thrust effects do not have to be

considered separately, because the thrust effects are fully represented between the density effects and the weight effects.
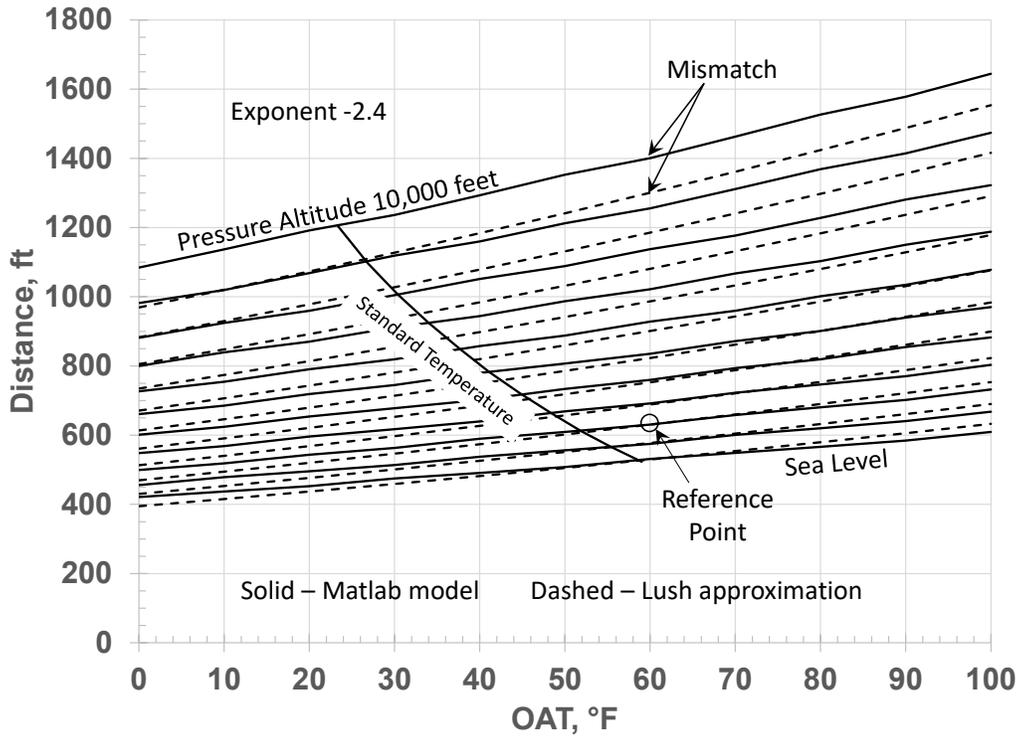

Fig. 2 Matlab model and Lush approximation for density effects, exponent = -2.4
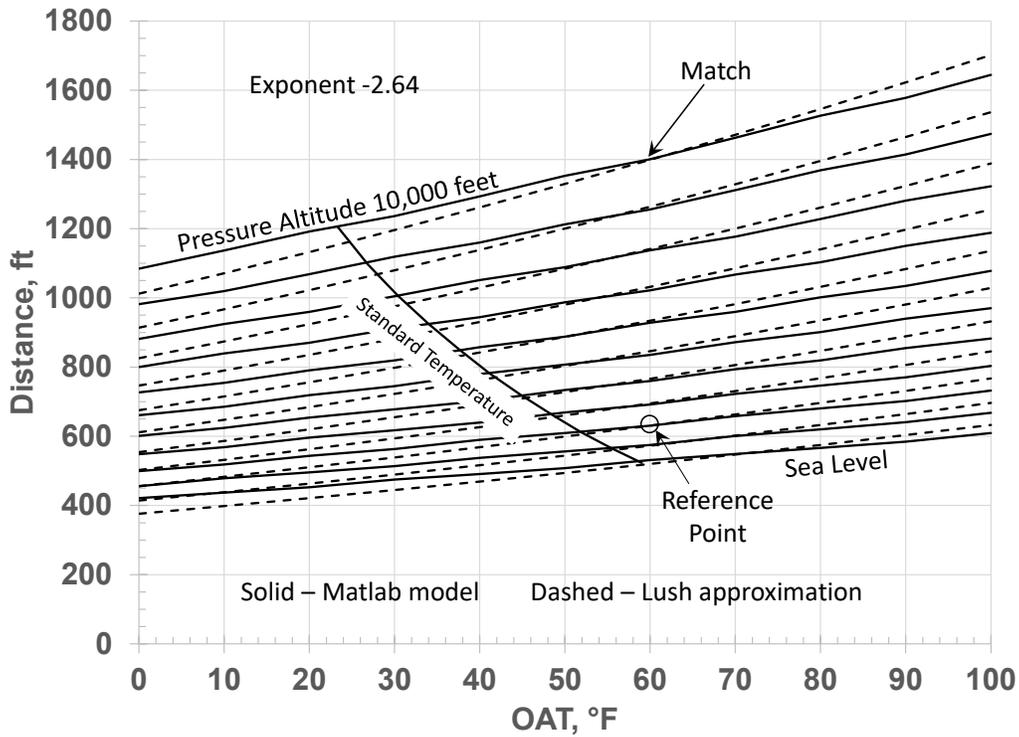

Fig. 3 Matlab model and Lush approximation for density effects, exponent = -2.64

B. Gross Weight Correction

The Lush correction for gross weight is given by (Eqn 6.328 [2])

$$S_W = \frac{s_{wt}}{\left(\frac{W_s}{W_t}\right)^{2.4}} \qquad (2)$$

where

s_{wt} ground roll distance at reference gross weight, zero wind, corrected for density
s_w ground roll distance at zero wind, corrected for density and gross weight
W_s gross weight at reference conditions
W_t gross weight at test conditions

The gross weight adjustment curves from the center of Fig. 1 are shown as solid lines in Fig. 4. Dotted lines for the Lush approximation are calculated from points starting at the reference gross weight of 2400 pounds, starting at ground roll distances every 100 feet from 500 feet to 1600 feet.
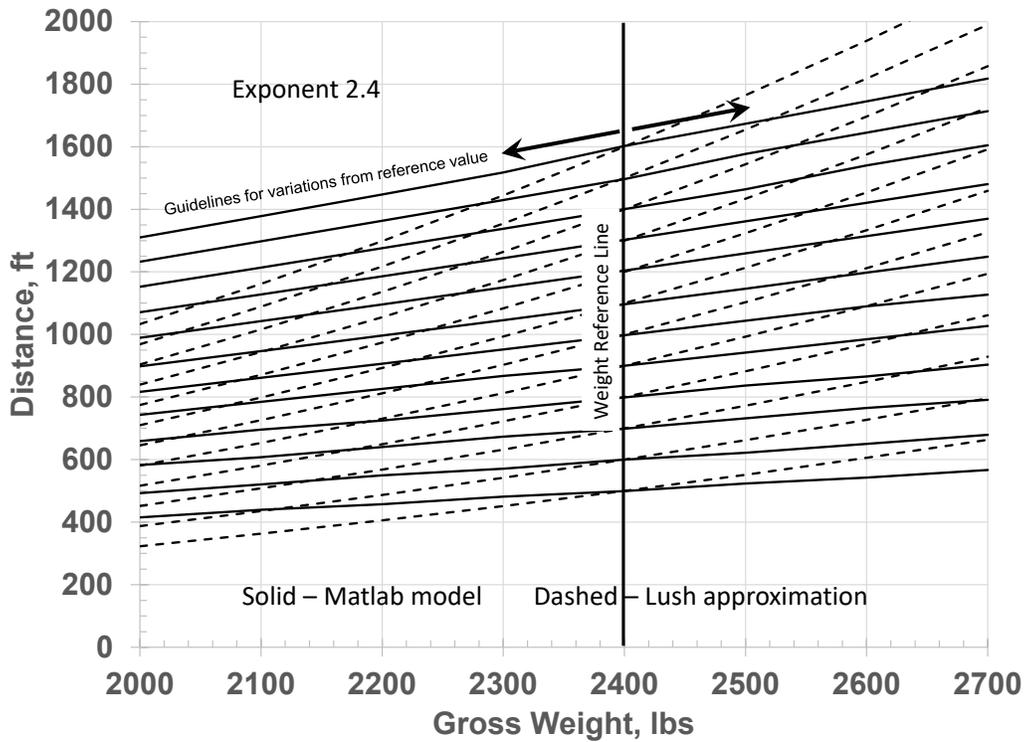

Fig. 4 Matlab model and Lush approximation for gross weight effects, exponent = 2.4

Clearly, the Lush approximation lines in Fig. 4 are not even close to parallel to the Matlab model lines. This can be traced to a difference in assumptions. In Lush's method, liftoff occurs at a specified lift coefficient. In the Matlab model, liftoff occurs at a specified calibrated airspeed, regardless of the lift coefficient required.

We can get a sense of where the Lush exponent comes from by looking at this takeoff ground roll equation from Ref. [6] in terms of a constant lift coefficient.

$$S_{LO} = \frac{1.44W^2}{g\rho_\infty S C_{L,max}\left\{T - \left[D + \mu_r(W - L)\right]_{ave}\right\}} \qquad (3)$$

This equation would seem to imply that the ground roll distance should be a function of the square of the gross weight, hence an exponent of 2.4 on the weight ratio. Looking into the derivation of this equation, the "1.44" factor indicates that this equation assumes liftoff at 1.2 times the stall speed. Stall speed is itself a function of gross weight.

However, if we look at the takeoff ground roll equation in terms of liftoff airspeed from which the Lush equations were derived [6], then we see a different weight relationship if liftoff airspeed is kept constant.

$$S_{LO} = \frac{\left(V_{LO}^2\right)(W/g)}{2\left\{T - \left[D + \mu_r(W - L)\right]_{ave}\right\}}$$ (4)

In this case, if the liftoff speed is arbitrarily set, as was done in the Matlab model, then the takeoff ground roll is approximately a function of weight to the first power. Adjusting the exponent in the weight equation to 1.1 gives Lush approximation curves that are almost indistinguishable from the Matlab model curves, as shown in Fig. 5.
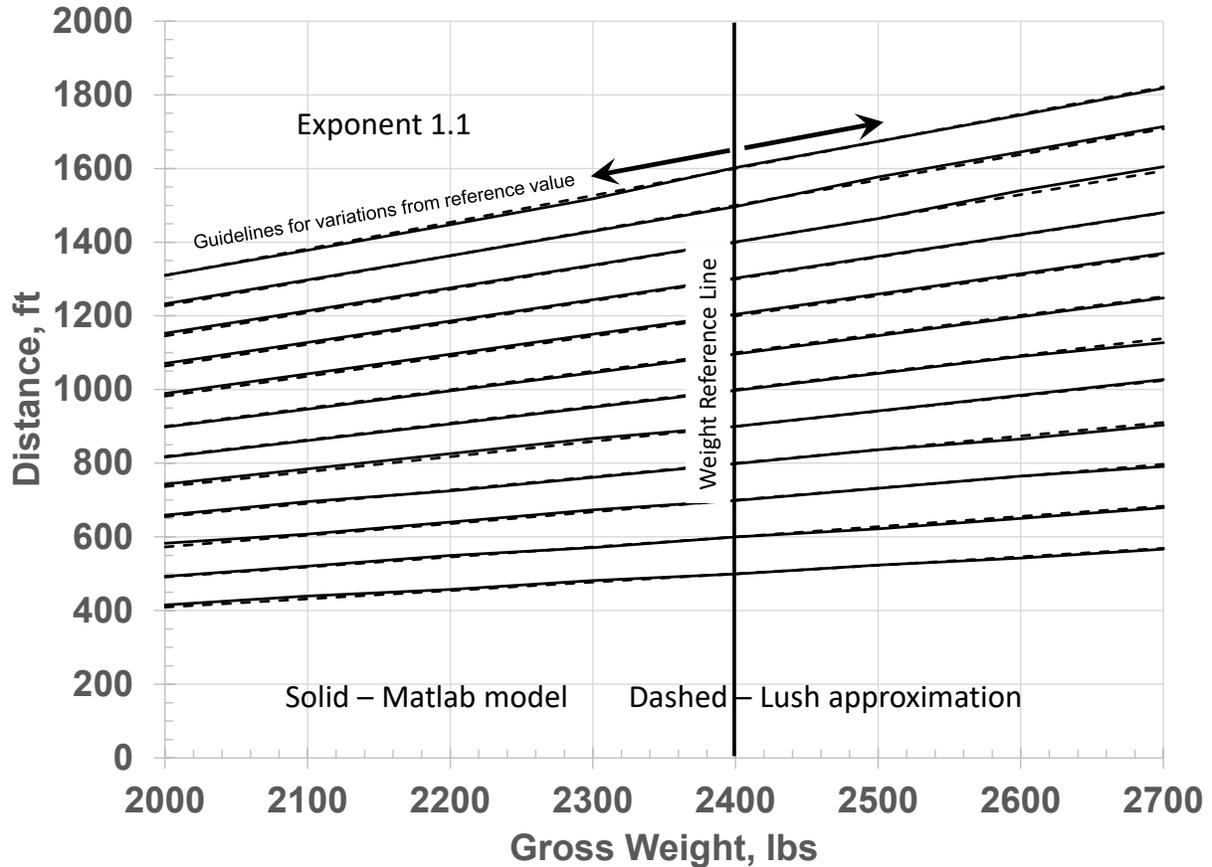
Fig. 5 Matlab model and Lush approximation for gross weight effects, exponent = 1.1

C. Wind Correction
The Lush correction for wind is given by Eqn 5 (Eqn 6.301 [2]). The empirical exponent is usually taken as 1.85 or 1.9 [3].

$$S_{level} = \frac{S_w}{\left(\frac{V_{to}+V_w}{V_{to}}\right)^{1.85}}$$ (5)

where

S_w	ground roll distance at zero wind, corrected for density and gross weight
S_{level}	ground roll distance on a level runway, corrected for density, gross weight, and headwind
V_{to}	liftoff ground speed
V_w	wind speed (headwind positive)

Lush created the equation above in terms of ground speed, which is easily measurable by cameras or other observation. However, the takeoff technique we are modeling is defined in terms of a liftoff airspeed, and a calibrated airspeed at that. Because ground speed plus headwind (headwind is positive) is true airspeed, we can recast the above equation in terms of true airspeed as

$$S_{level} = \frac{S_w}{\left(\frac{V_t}{V_t - V_w}\right)^{1.85}} \tag{6}$$

where

S_w	ground roll distance at zero wind, corrected for density and gross weight
S_{level}	ground roll distance on a level runway, corrected for density, gross weight, and headwind
V_t	liftoff true airspeed
V_w	headwind speed (headwind positive)

The wind adjustment curves from the right of Fig. 1 are shown as solid lines in Fig. 6. Curves for tailwinds are shown as negative headwinds. Dotted lines for the Lush approximation are calculated from points starting at the reference wind of zero knots, starting at takeoff roll distances every 100 feet from 500 feet to 1600 feet.
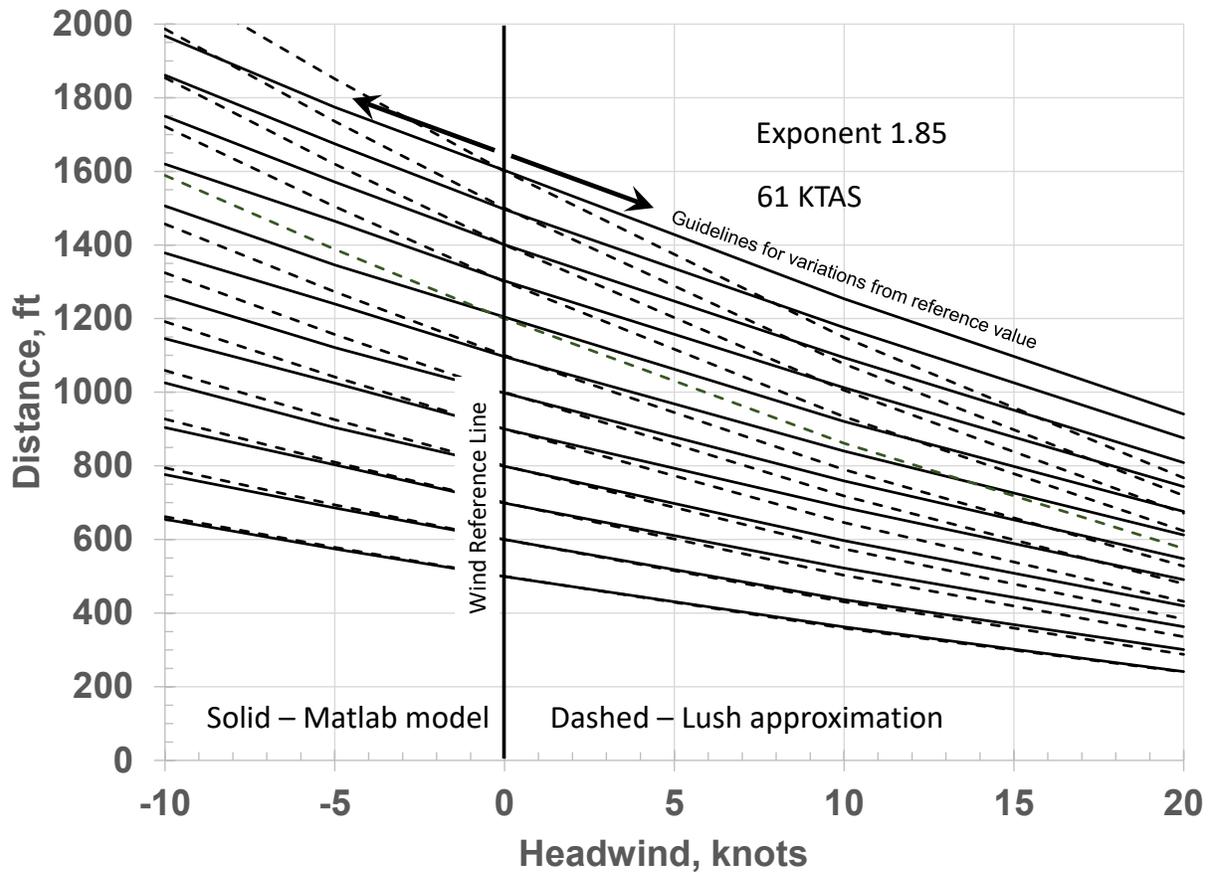

Fig. 6 Matlab model and Lush approximation for wind effects, exponent = 1.85, liftoff at 61 KTAS

The comparison between the Matlab model and the Lush approximation in Fig. 6 is made at a liftoff true airspeed of 61 KTAS. This airspeed is a good representation at sea level on a standard day. As the density altitude increases (higher pressure altitude and/or higher temperature) the true airspeed corresponding to 61 KCAS will increase, which makes it difficult to compare these two methods on a simple plot. In Fig. 6, the dashed lines are very parallel to the solid lines at lower takeoff distances, like those seen when operating near a density altitude near sea level. However, at longer takeoff distances, as would be represented by higher density altitudes, the dashed lines are not parallel to the solid lines. At these conditions, though, the liftoff true airspeed would be higher than 61 KTAS. To represent this

case, Fig. 7 is drawn for a liftoff true airspeed of 70 KTAS, which would be the liftoff airspeed around 9000 feet density altitude.
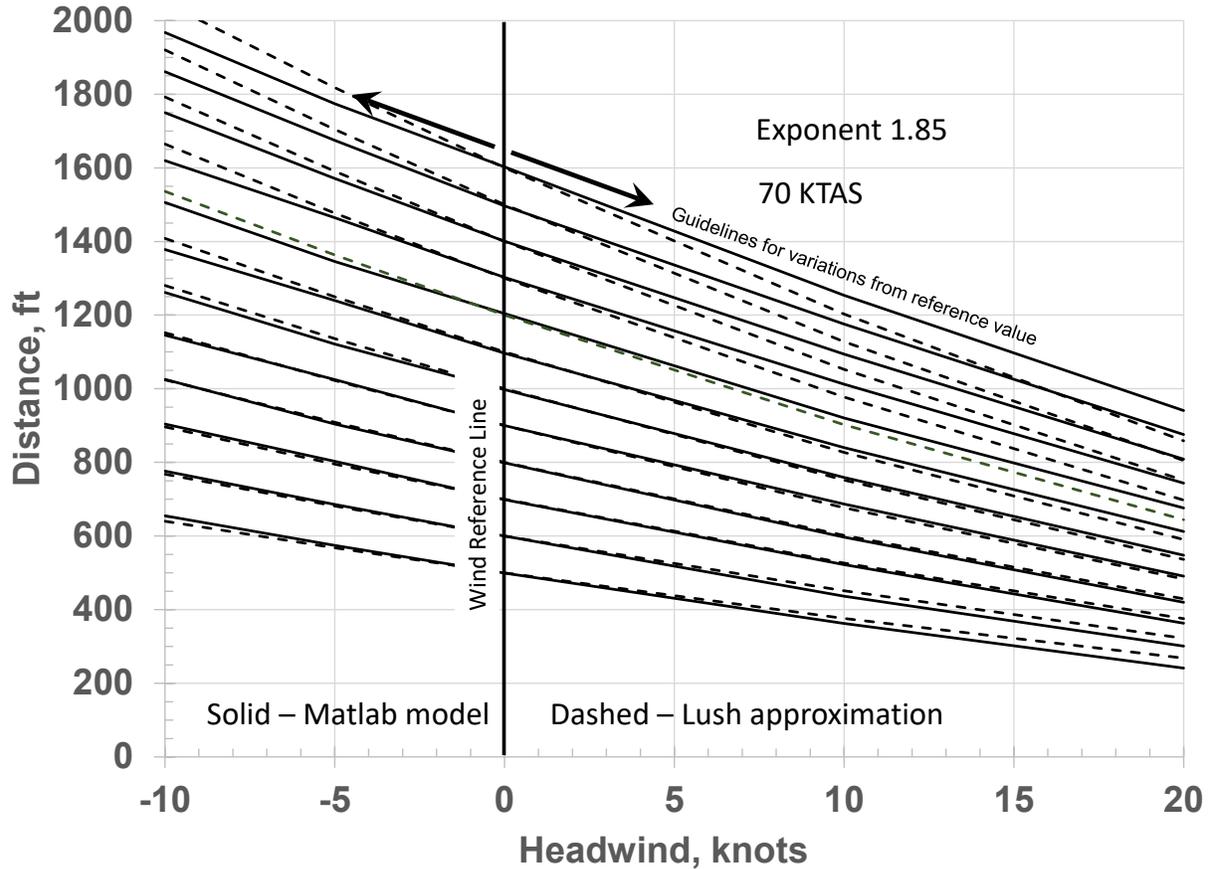

Fig. 7 Matlab model and Lush approximation for wind effects, exponent = 1.85, liftoff at 70 KTAS

The nominal takeoff distance at 9000 feet density altitude is about 1100 feet. Looking at Fig. 7 around 800 to 1200 feet distance, the dashed curves are now relatively parallel to the solid curves. This equation with its exponent of 1.85 seems to work acceptably if the true airspeed is used, which can be easily calculated as 61 KCAS divided by the square root of the density ratio sigma. In further support, in Ref. [3] Lush states "It will be seen that the cases differ little up to very high wind speeds, and also that the empirical formula agrees excellently. As its form is very convenient it is, therefore, proposed to retain it."

Even though the results in Fig. 6 and 7 look good (errors smaller than 80 feet ground roll at 20 knot headwinds and much smaller at low wind speeds), care should be taken when computing for tailwinds or headwinds over 10 knots. FAA Advisory Circular AC25-7C [7] states that 10 knots should be considered the maximum acceptable headwind for takeoff and landing flight tests. FAA Advisory Circular AC23-8C [8] repeats this statement and adds "At higher wind velocities, the data may be unreliable due to wind variations and unsmooth flight conditions." Stated another way, at higher wind speeds the assumption that the wind velocity is constant becomes suspect. Taking off with anything more than a light tailwind should be avoided whenever possible. A common technique in the military for takeoff planning is to use the no-wind distance and take either 50% of the headwind or 150% of a tailwind into account. This technique will eliminate most of the ambiguity high winds might add to the takeoff distance.

D. Combined Correction

Thus, the final Lush approximation for this Bearhawk is given by

$$s_{level} = \frac{s_{std}}{\left(\frac{\sigma_s}{\sigma_t}\right)^{-2.64} \left(\frac{W_s}{W_t}\right)^{1.1} \left(\frac{V_t}{V_t - V_w}\right)^{1.85}} \tag{7}$$

with constant values for this model defined as

s_{std}	630.5 feet
σ_s	0.928021
W_s	2400 pounds
V_t	True Airspeed corresponding to 61 KCAS $\left(\frac{61\ KCAS}{\sqrt{\sigma_t}}\right)$

A spreadsheet app was created using these approximations for convenient takeoff distance predictions.

III. Lush Approximation Error Analysis

So, if the Lush methods are just approximations, how much error do they introduce? An exhaustive comparison of every possible condition (11 pressure altitudes * 11 temperatures * 8 gross weights * 5 wind speeds) would be 4840 simulation runs, which seemed a bit excessive. Instead, a Monte Carlo simulation approach was used.

A uniform distribution was assumed for pressure altitude, temperature, gross weight, and headwind within the ranges previously calculated. For 300 random combinations, the takeoff ground roll distance was calculated using the Matlab model [1] and using the Lush approximations. The difference of these results was analyzed.

To confirm that 300 samples was sufficiently large, the standard deviation of several Monte Carlo runs at varying sample sizes of 25, 50, 100, 200, and 300 are shown in Fig. 8. A sample size of 25 is clearly not enough with a large variance in standard deviation. However, with a sample size of 300 the standard deviation seems to have settled down.

Pressure Altitude 0 to 10,000 feet   Gross Weight 2000 to 2700 pounds
OAT 0 to 100°F   Headwind -10 to 20 knots
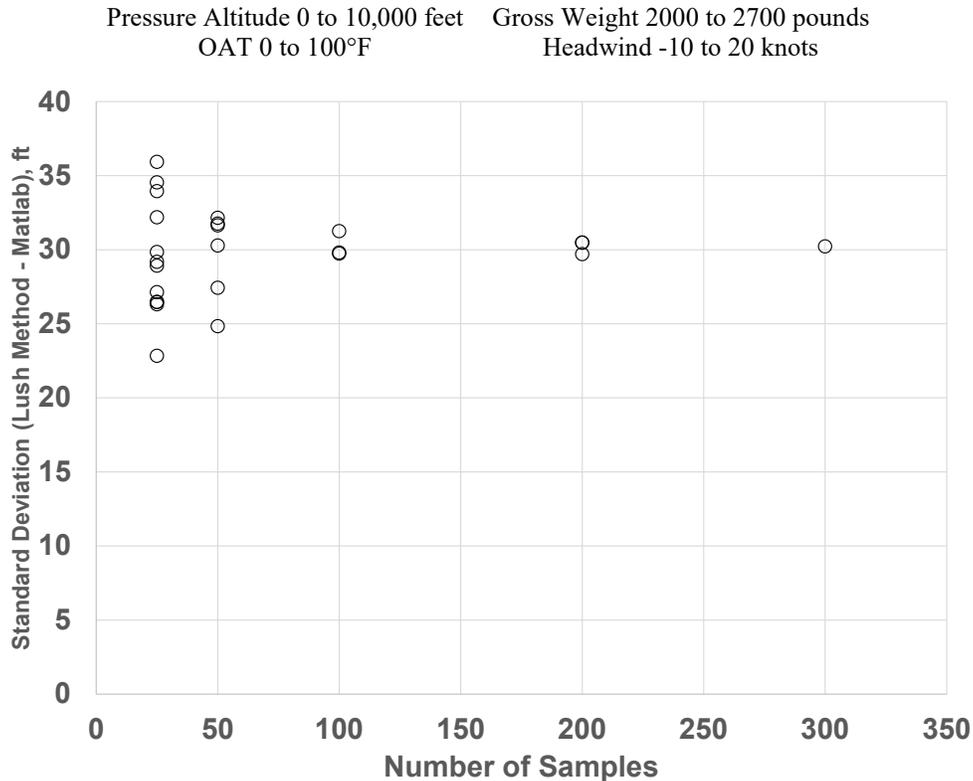
Fig. 8 Standard Deviation of different sample sizes

Fig. 9 shows a histogram of the errors between the Lush approximation takeoff ground roll and the Matlab model ground roll from this Monte Carlo data set. The error seems to be mostly between -49 feet (Lush predicts short) and 21 feet (Lush predicts long). Errors of these sizes are assumed to be well within the uncertainty of actual takeoff ground rolls.

Pressure Altitude 0 to 10,000 feet   Gross Weight 2000 to 2700 pounds
OAT 0 to 100°F   Headwind -10 to 20 knots

Fig. 9 Lush approximation error histogram

The histogram of the error, shown in Fig. 9, deviates a little from a traditional normal distribution.   For more insight, Fig. 10 plots the errors against the headwind value.

Pressure Altitude 0 to 10,000 feet   Gross Weight 2000 to 2700 pounds
OAT 0 to 100°F   Headwind -10 to 20 knots

Fig. 10 Wind effects on Lush approximation error distribution

Within the recommended wind zone of zero to ten knots, the errors are small, generally between 60 feet short and 40 feet long. Since the gros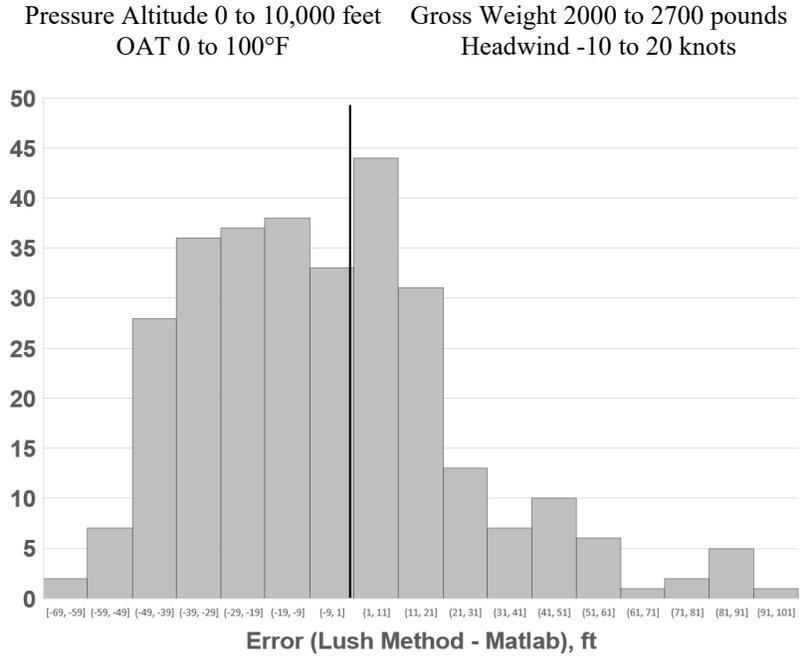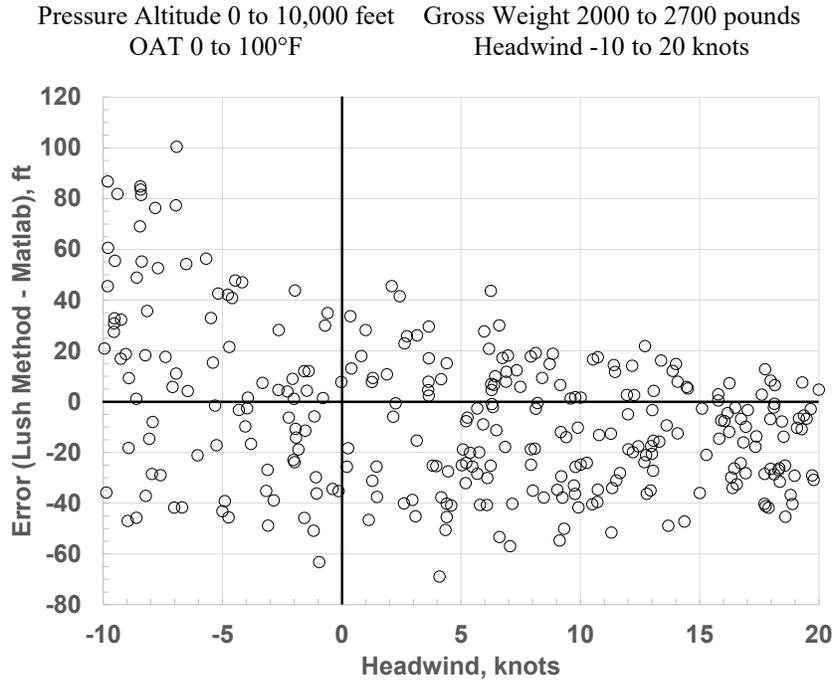s weight approximation (Fig. 5) is pretty good and the wind correction is reasonable within this range, it is suspected that the tendency toward negative errors is because of the density correction. The density correction (Fig. 3) was set up to be good around comfortable flying temperatures (50 to 75°F), but the Monte Carlo simulation used a uniform temperature distribution from 0 to 100°F, which would skew the results to lower temperatures where the Lush approximation predicts a shorter takeoff run. To confirm this assertion, Fig. 11 shows the errors only for the Monte Carlo points with temperatures between 50 to 75°F. These errors are relatively evenly distributed around zero and have magnitudes generally less than ±20 feet.

Pressure Altitude 0 to 10,000 feet   Gross Weight 2000 to 2700 pounds
OAT 50 to 75°F   Headwind -10 to 20 knots

Fig. 11 Wind effects on Lush approximation error distribution, OAT 50-75F

Fig. 12 shows the errors only for the Monte Carlo points with temperatures between 0 to 50°F. Fig. 3 shows that the Lush predicted ground rolls would expect to be shorter (negative errors) at these temperatures. Figure 12 shows that almost all of the errors are negative.

Pressure Altitude 0 to 10,000 feet   Gross Weight 2000 to 2700 pounds
OAT 0 to 50°F   Headwind -10 to 20 knots

Fig. 12 Wind effects on Lush approximation error distribution, OAT 0-50F

Fig. 13 shows the errors only for the Monte Carlo points with temperatures between 75 to 100°F. Fig. 3 shows that the Lush predicted ground rolls would expect to be longer (positive errors) at these temperatures. Fig. 13 shows that almost all of the errors are positive.
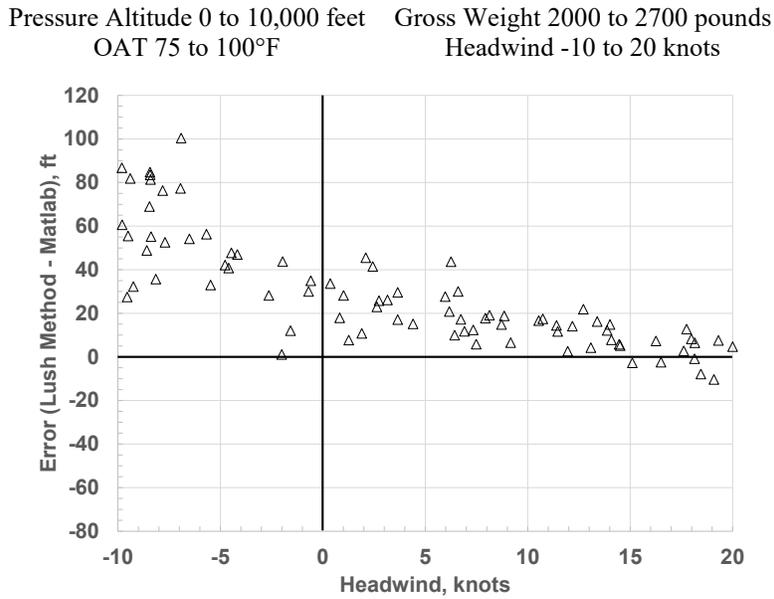
Pressure Altitude 0 to 10,000 feet   Gross Weight 2000 to 2700 pounds
OAT 75 to 100°F   Headwind -10 to 20 knots

Fig. 13 Wind effects on Lush approximation error distribution, OAT 75-100F

Fig. 10 also shows that the right-side tail of Fig. 9 seems to be concentrated at large values of tailwinds, where the wind correction is expected to be worse. This is acceptable since takeoff would normally be avoided in these conditions.

To get a sense for the magnitude of these errors as related to the Matlab modeled ground roll, Fig. 14 plots the errors between the Lush approximation and the Matlab calculated ground roll against the Matlab calculated ground roll.
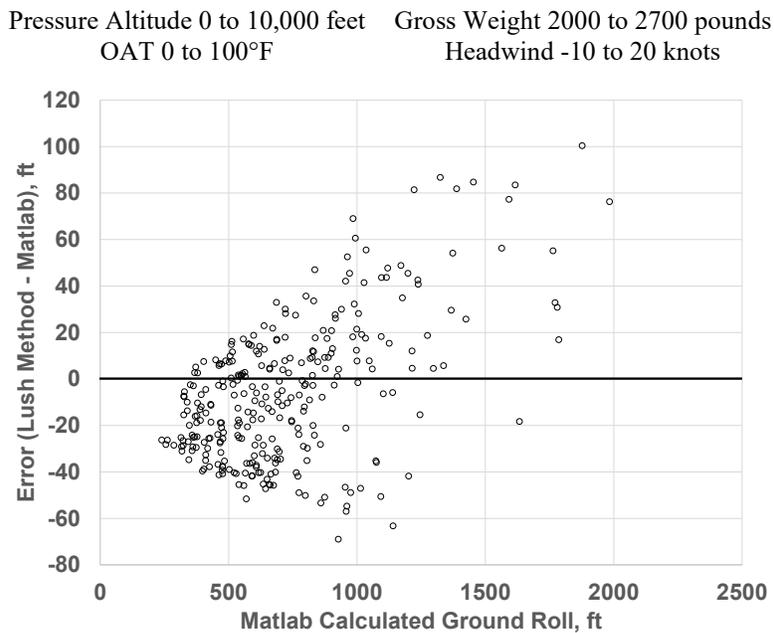
Pressure Altitude 0 to 10,000 feet   Gross Weight 2000 to 2700 pounds
OAT 0 to 100°F   Headwind -10 to 20 knots

Fig. 14 Lush approximation error analysis

Fig. 14 shows that most takeoffs are less than 1000 feet ground roll, and the error introduced by using the Lush approximation is less than about 6 per cent. This is well within the 50 per cent that ASI recommends buffering the calculated takeoff distance. The larger errors that Fig. 10 blames on tailwinds indeed plot out as longer ground roll distances in Fig. 14.

IV. Conclusion

A simplified takeoff model, based on the methods of Ken Lush, was created that is suitable for running on a simple spreadsheet on a portable device, such as a smartphone or tablet. Within the limits of the Lush method, the exponents in the equations were adjusted to give the best possible agreement with the Matlab model, concentrating on conditions where takeoffs would most likely occur. The final Lush approximation for this Bearhawk is given by

$$s_{level} \ = \ \frac{s_{std}}{\left(\frac{\sigma_s}{\sigma_t}\right)^{-2.64} \left(\frac{W_s}{W_t}\right)^{1.1} \left(\frac{V_t}{V_t - V_w}\right)^{1.85}} \tag{7}$$

where

s_{level}	ground roll distance on a level runway, corrected for density, gross weight, and headwind
σ_t	density ratio at test condition
W_t	gross weight at test condition
V_w	wind speed (headwind positive)

with constant values for this model defined as

s_{std}	630.5 feet
σ_s	0.928021
W_s	2400 pounds
V_t	True Airspeed corresponding to 61 KCAS $\left(\frac{61\ KCAS}{\sqrt{\sigma_t}}\right)$

The residual differences between the Lush approximation and the Matlab model were analyzed and found to be acceptable and within the expected uncertainty of normal takeoff ground roll distances. Thus, the Lush approximation is suitable for real-time flight planning.

References

[1] Erb, R. E. and Major, K. B., "Takeoff Chart Development for a Homebuilt Airplane by Numerical Simulation," AIAA SciTech 2026 (to be published).
[2] Herrington, R. M., Major, USAF, et al, "Flight Test Engineering Handbook," AF Technical Report 6273, AFFTC, Edwards AFB, California, 1966.
[3] Lush, K. J., "Standardization of Take-Off Performance Measurements for Airplanes," Technical Note R-12, AFFTC, Edwards AFB, California.
[4] Erb, R. E., "A Low Cost Method for Generating Takeoff Ground Roll Charts from Flight Test Data," http://www.erbman.org/sfte/TOPAPER.pdf , Society of Flight Test Engineers Annual Symposium, 1996.
[5] "Safety Advisor Mastering Takeoffs & Landings," https://download.aopa.org/asi/flightreview/PositiveAircraftControl.pdf, Aircraft Owner's and Pilot's Association Air Safety Institute, 2008.
[6] Anderson, J. D. Jr., *Introduction to Flight* Third Edition, McGraw-Hill Book Company, New York, 1989.
[7] AC 25-7C, "Flight Test Guide For Certification Of Transport Category Airplanes," https://www.faa.gov/documentLibrary/media/Advisory_Circular/AC%2025-7C%20.pdf , 2012.
[8] AC 23-8C, "Flight Test Guide for Certification of Part 23 Airplanes," https://www.faa.gov/regulations_policies/advisory_circulars/index.cfm/go/document.information/documentid/1019676 , 2011.